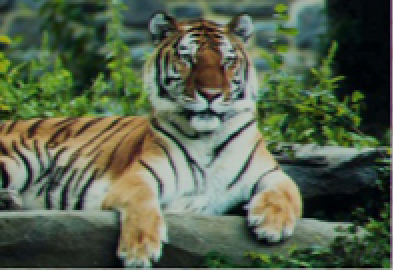
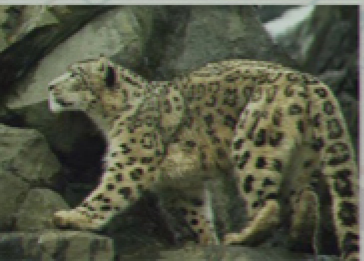


Why can you find animals with spotted coats and striped tails but no animals with striped bodies and spotted tails?



Keith Devlin

Stanford University

The answer involves mathematics

The answer involves mathematics

– a ***LOT*** of mathematics!

What is mathematics?

- Pre-500 BC: Use of arithmetic, some geometry and trigonometry.
- 500 BC–300 AD: Detailed study of number and shape.
- 17th Century: the study of number, shape, and motion (calculus).
- 20th Century: the study of patterns.

What kinds of pattern?

- Counting (numbers, arithmetic)
- Numbers (number theory)
- Shape (geometry)
- Measuring (e.g. trigonometry)
- Motion and change (calculus)
- Putting things together (algebra)
- Chance events (probability theory)
- etc.

How do we ***see*** these kinds of pattern?

Mathematics is a ***language*** for describing abstract patterns.

When we use it to do that, it gives rise to the "***Science of*** (abstract) ***Patterns***".

Sometimes with our eyes …

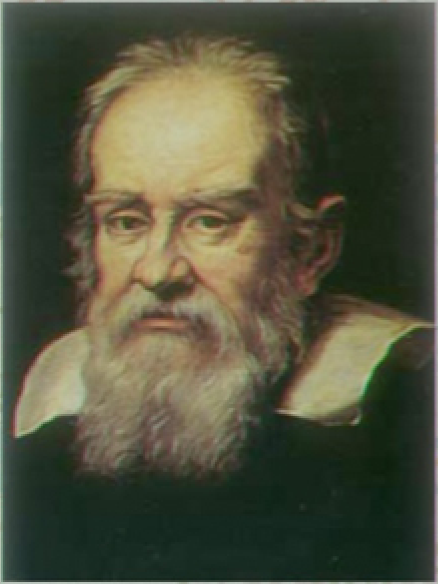
but mostly with our minds – through mathematics.

What do we use this mathematical language for?

To understand our world
(and ourselves)
and use that understanding
to do things in the world.

Who is this?

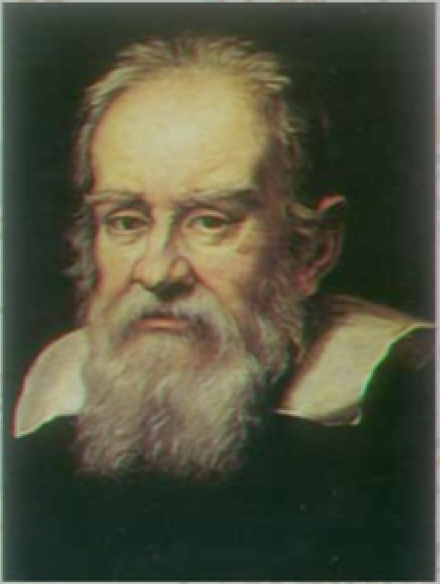
and what is he famous for?



Galileo (1564 – 1642)

The inventor of modern science

"To understand the universe, you have to understand the language in which it is written. That language is mathematics."

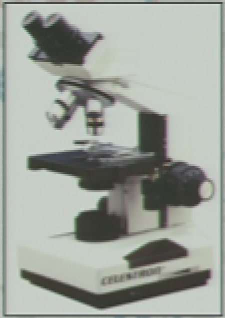


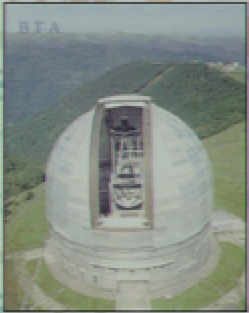
How does this language called mathematics help us to understand our world?

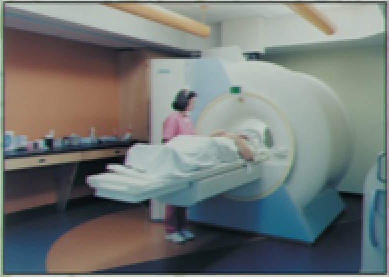
It makes the invisible visible

Making the invisible visible

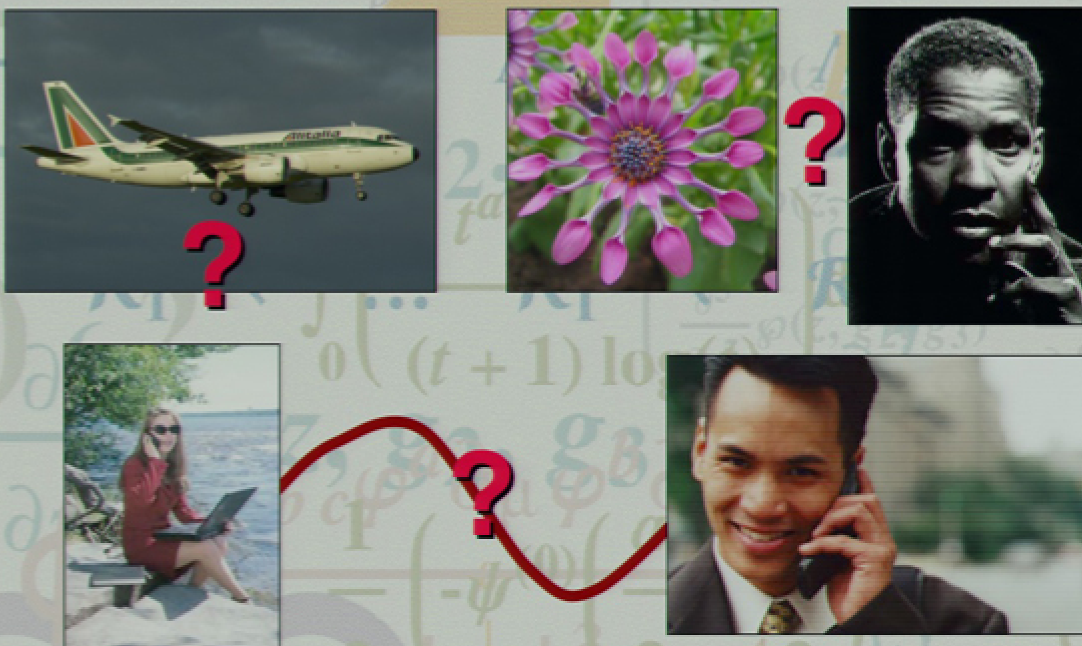
More technologies that make the invisible visible







Making the invisible visible using mathematics



Why are there all those symbols?

$$A_n^2 = \frac{1}{4}\epsilon_{ijk}\, PQ_j^n\, PQ_k^{n+1}\, \epsilon_{ilm}\, PQ_l^n\, PQ_m^{n+1},$$

where ϵ_{ijk} is the permutation symbol. Using the Kronecker delta δ_{ij}, and using $\frac{\partial P_i}{\partial P_q} = \delta_{iq}$ as well as $\nabla = \partial/\partial P_q$, we derive:

$$\begin{aligned}\frac{\partial A_i^2}{\partial P_q} &= 2A_i\frac{\partial A_i}{\partial P_q}\\ &= \frac{1}{4}\epsilon_{ijk}\epsilon_{ilm}\Big[-\delta_{jq}\, PQ_k^{n+1}\, PQ_l^n PQ_m^{n+1} - \delta_{kq}\, PQ_j^n\, PQ_l^n\, PQ_m^{n+1}\\ &\quad -\delta_{lq}\, PQ_j^n\, PQ_k^{n+1}\, PQ_m^{n+1} - \delta_{mq}\, PQ_j^n\, PQ_k^{n+1}\, PQ_l^{n+1}\Big]\end{aligned}$$

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Consequently:

$$\frac{\partial A_i}{\partial P} = \frac{1}{4A_i}\Big((PQ^{n+1}\cdot Q^{n+1}Q^n)\, PQ^n + (PQ^n\cdot Q^nQ^{n+1})\, PQ^{n+1}\Big). \tag{15}$$

Using Equ. (13), we find:

$$\frac{\nabla A}{2A} = \frac{1}{2A}\sum_i \frac{\partial A_i}{\partial P} \tag{16}$$

You need an abstract notation to describe abstract structures and patterns precisely.

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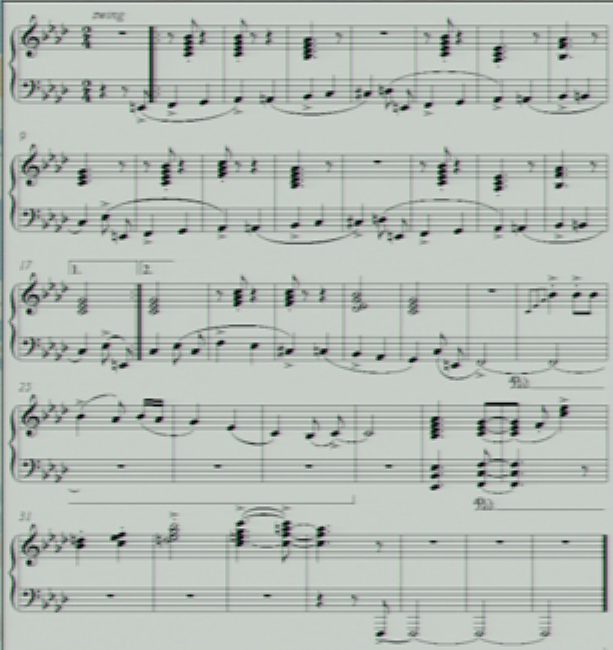

What are these abstract symbols good for?

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p \quad \text{Euler's equation}$$

$$\mathbf{v} = -\nabla\phi \quad \text{so } \nabla\times\mathbf{v} = 0 \quad \text{irrotational}$$

$$\mathbf{F} = -\nabla\Omega \quad \text{conservative}$$

$$\rho = \text{const. or } f(p) \quad \text{incompressible}$$

$$\frac{\partial}{\partial t}\left(-\nabla\phi\right) + \nabla\phi\cdot\nabla\nabla\phi = -\nabla\Omega - \frac{1}{\rho}\nabla p$$

$$\nabla\left[-\frac{\partial\phi}{\partial t} + \frac{v^2}{2} + \Omega + \frac{p}{\rho}\right] = 0$$

$$-\frac{\partial\phi}{\partial t} + \frac{v^2}{2} + \Omega + \frac{p}{\rho} = C$$

$$\frac{v^2}{2} + \Omega + \frac{p}{\rho} = C \quad \text{Bernoulli's equation}$$

Bernoulli's Equation

They let us “see” one of the forces that keep this in the air

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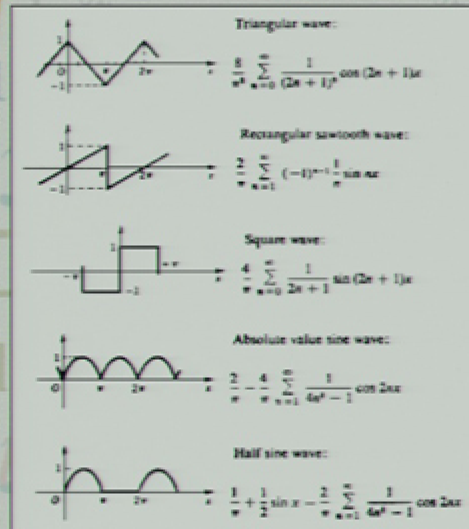
Bernoulli's Equation

They let us "see" one of the forces that keep this in the air

How does this store music?

Using this mathematics



Fourier series

What do these equations help us to understand?

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) + \nabla p = \mu\Delta\mathbf{u} + \mathbf{f},$$

$$\nabla\cdot\mathbf{u} = 0,$$

$$\rho(\mathbf{x},t) = \int M(q,r,s)\,\delta\left(\mathbf{x} - \mathbf{X}(q,r,s,t)\right)\mathrm{d}q\,\mathrm{d}r\,\mathrm{d}s,$$

$$\mathbf{f}(\mathbf{x},t) = \int \mathbf{F}(q,r,s,t)\,\delta\left(\mathbf{x} - \mathbf{X}(q,r,s,t)\right)\mathrm{d}q\,\mathrm{d}r\,\mathrm{d}s,$$

$$\frac{\partial \mathbf{X}}{\partial t}(q,r,s,t) = \mathbf{u}\left(\mathbf{X}(q,r,s,t),t\right)$$

$$= \int \mathbf{u}(\mathbf{x},t)\,\delta\left(\mathbf{x} - \mathbf{X}(q,r,s,t)\right)\mathrm{d}\mathbf{x},$$

$$\mathbf{F} = -\frac{\wp E}{\wp \mathbf{X}}.$$

These equations help us understand how the human heart works

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) + \nabla p = \mu\Delta\mathbf{u} + \mathbf{f},$$

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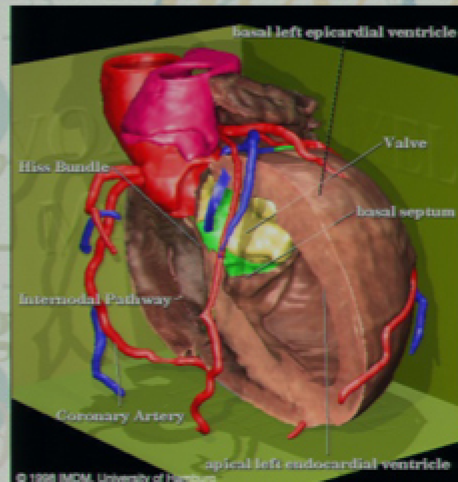


What do we see with this?

$$\dot{x} = f_1 = u$$

$$\dot{u} = f_2 = 2v + x - \frac{\mu(-1+x+\mu)}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{\frac{3}{2}}} - \frac{(1-\mu)(x+\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{\frac{3}{2}}}$$

$$\dot{y} = f_3 = v$$

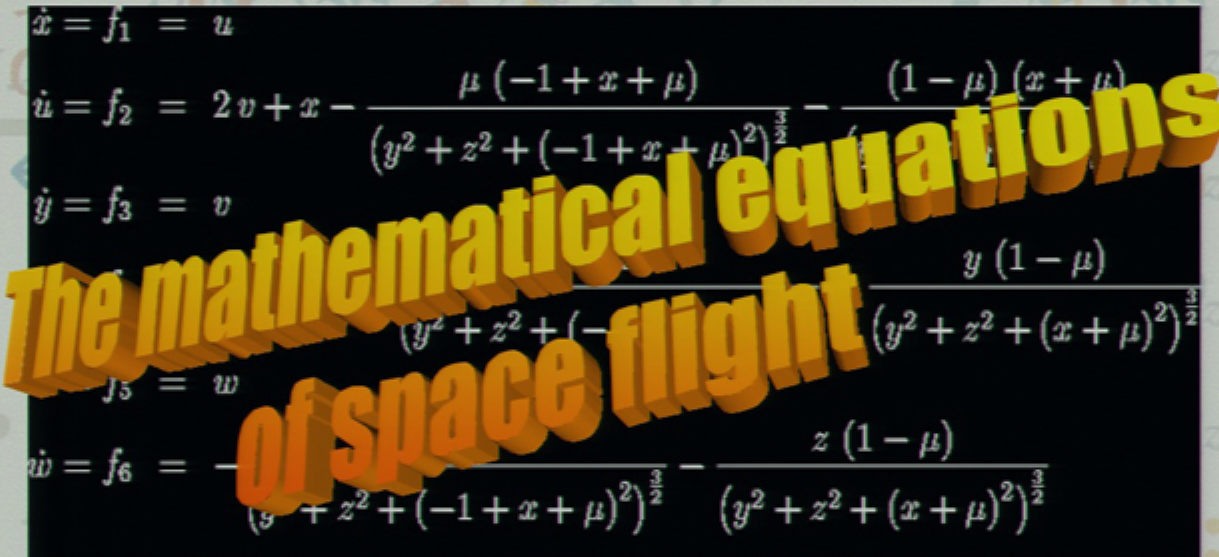
$$\dot{v} = f_4 = -2u + y - \frac{y\,\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{\frac{3}{2}}} - \frac{y(1-\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{\frac{3}{2}}}$$

$$\dot{z} = f_5 = w$$

$$\dot{w} = f_6 = -\frac{z\,\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{\frac{3}{2}}} - \frac{z(1-\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{\frac{3}{2}}}$$

This describes the forces that act on a body in outer space

What do we see with this?



This describes the forces that act on a body in outer space

Who uses these equations?

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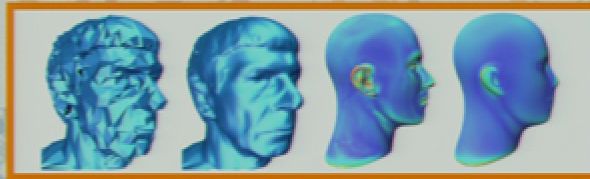

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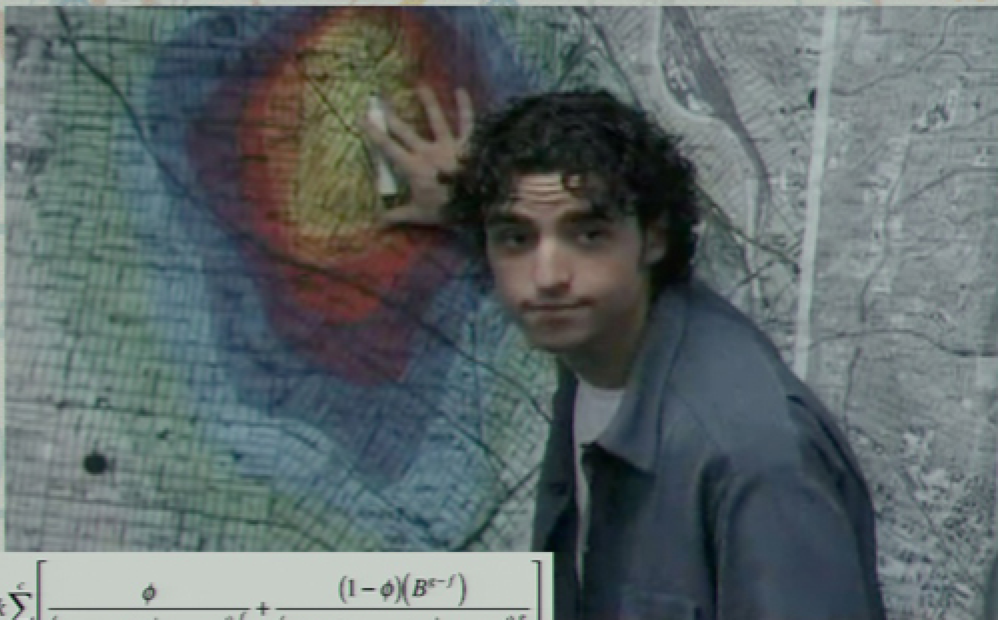
What is this math used for?

$$p_{ij} = k\sum_{n=1}^{c}\left[\frac{\phi}{\left(\left|x_i - x_n\right| + \left|y_j - y_n\right|\right)^f} + \frac{(1-\phi)\left(B^{g-f}\right)}{\left(2B - \left|x_i - x_n\right| - \left|y_j - y_n\right|\right)^g}\right]$$

Catching criminals

$$p_{ij} = k\sum_{n=1}^{c}\left[\frac{\phi}{\left(|x_i - x_n| + |y_j - y_n|\right)^f} + \frac{(1-\phi)\left(B^{g-f}\right)}{\left(2B - |x_i - x_n| - |y_j - y_n|\right)^g}\right]$$

NUMB3RS: First ever episode



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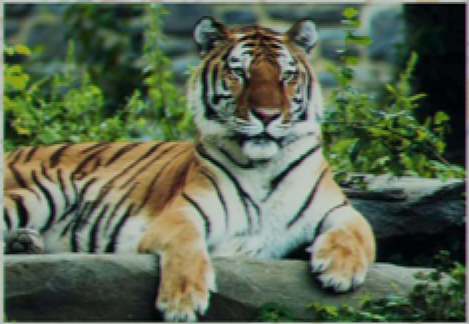
My examples show patterns of:

- Forces keeping and aircraft in the air
- Musical sounds (iPod)
- The human heartbeat
- Forces acting on a spacecraft
- The structure of things we see (movie graphics)
- Criminal behavior

Time to look at …

Animal coat patterns



Some animals don't have patterns

Some do have patterns

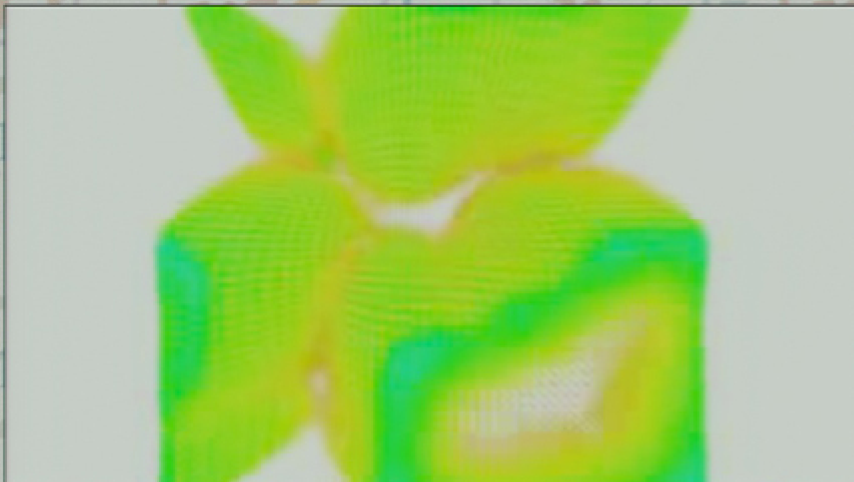
What are the most interesting patterns?

- We see the skin "pattern" with our eyes.
- Can we see use mathematics to see (with our minds) ***what creates*** those patterns of spots and stripes?
- Can we see ***nature's invisible pattern***?

How do the coat patterns arise?

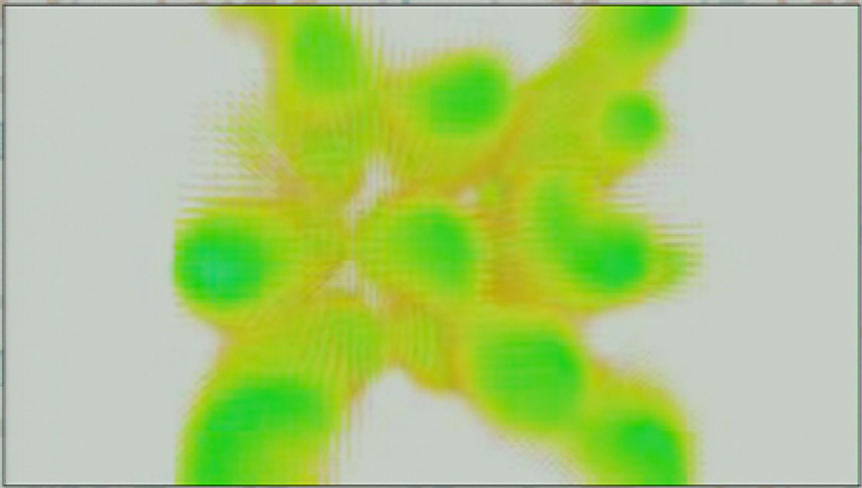
- Skin color is caused by a chemical called melanin
- The skin pattern is a result of different concentrations of melanin
- What makes the melanin concentrate the way it does?
- The basic mechanism is believed to be a "reaction-diffusion" process

Reaction-diffusion process



Computer simulation

Reaction-diffusion process



Computer simulation

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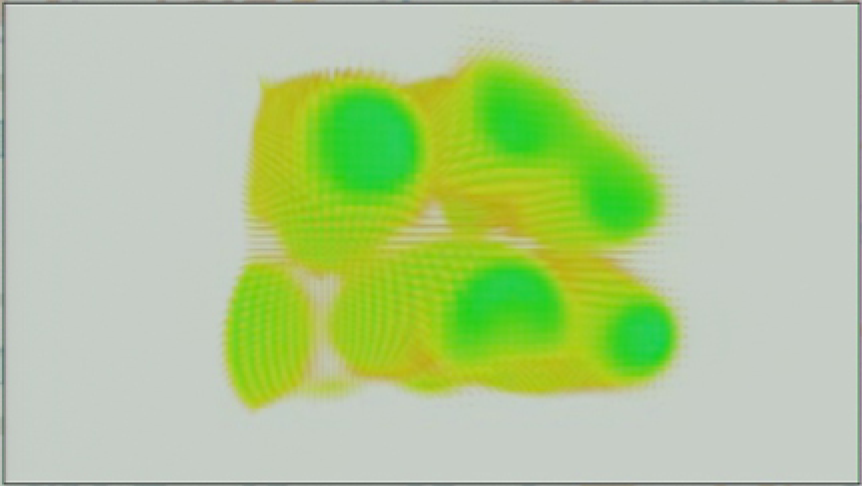
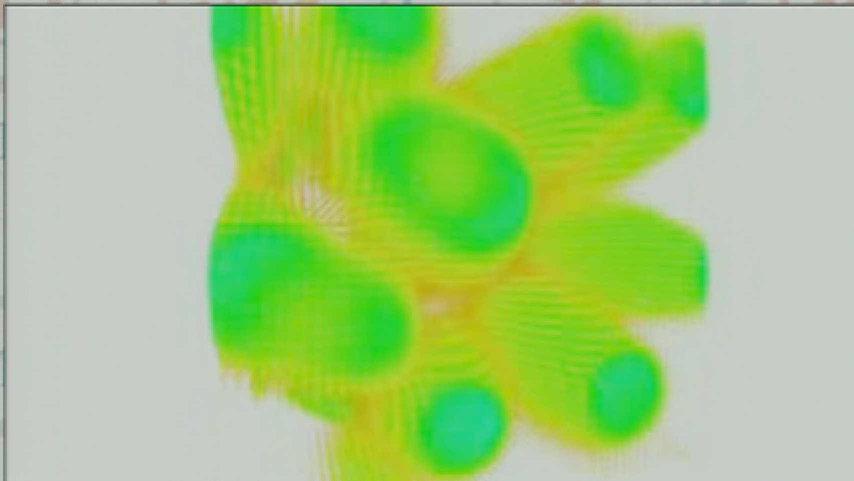


Computer simulation

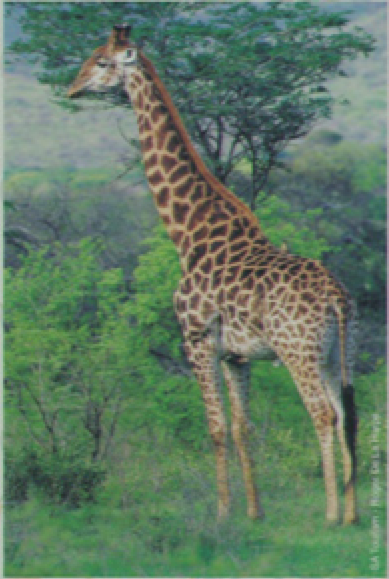
Reaction-diffusion process



Computer simulation

How we think it works

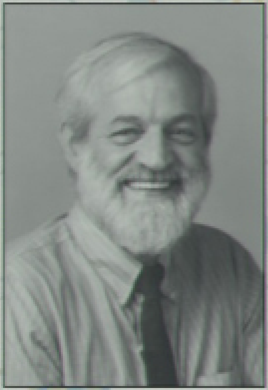
- Melanin production in the skin is initiated or sped-up by an "activator" chemical, and slowed down or stopped by an "inhibitor" chemical.
- The activator and inhibitor create a reaction-diffusion process.
- The inhibitor spreads faster.



- The initial distribution of the activator and inhibitor is random.

What makes the patterns different?

- The area and shape of the skin during the reaction.
- Small areas allow no space for the diffusion, so there is no pattern.
- With a large area the inhibitor eventually occupies the entire area, so again no pattern.
- Thus mice and elephants have neither stripes or spots.



James Murray

- In a long, thin rectangular area, the inhibitor and activator will form alternating bands, so you get stripes.
- In a squarish area, the inhibitor will surround areas of activator so you get spots.

When it happens

For most creatures, the key reaction-diffusion process takes place during the embryonic stage.

So their final coat pattern depends on the area and shape of the embryo, not the adult creature.

Animal coat patterns

Animal coat patterns

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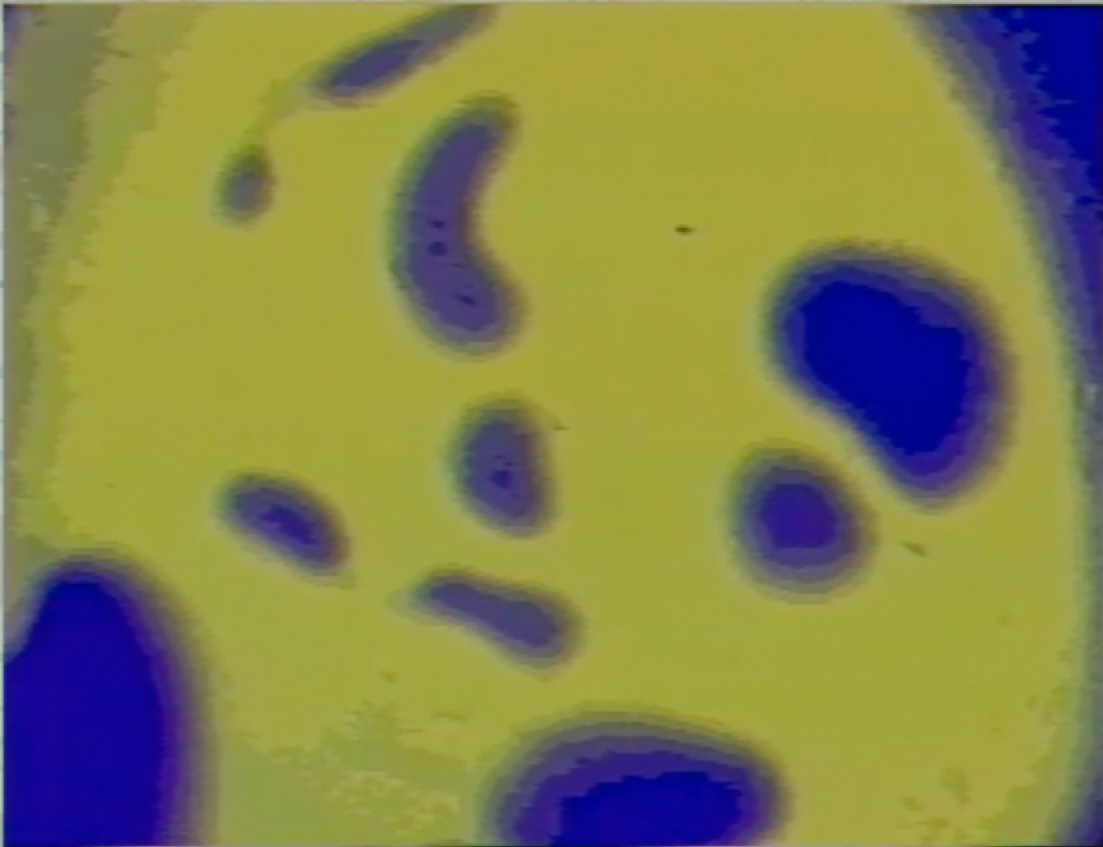
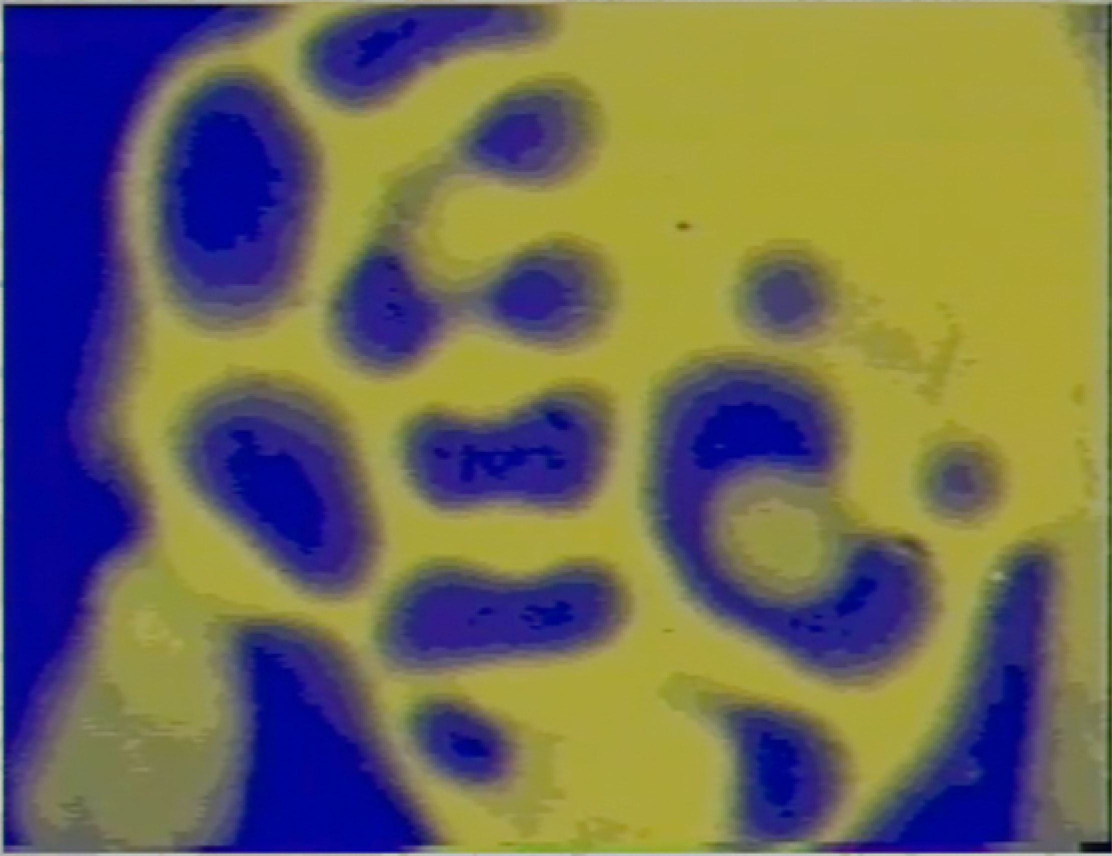


Animal coat patterns



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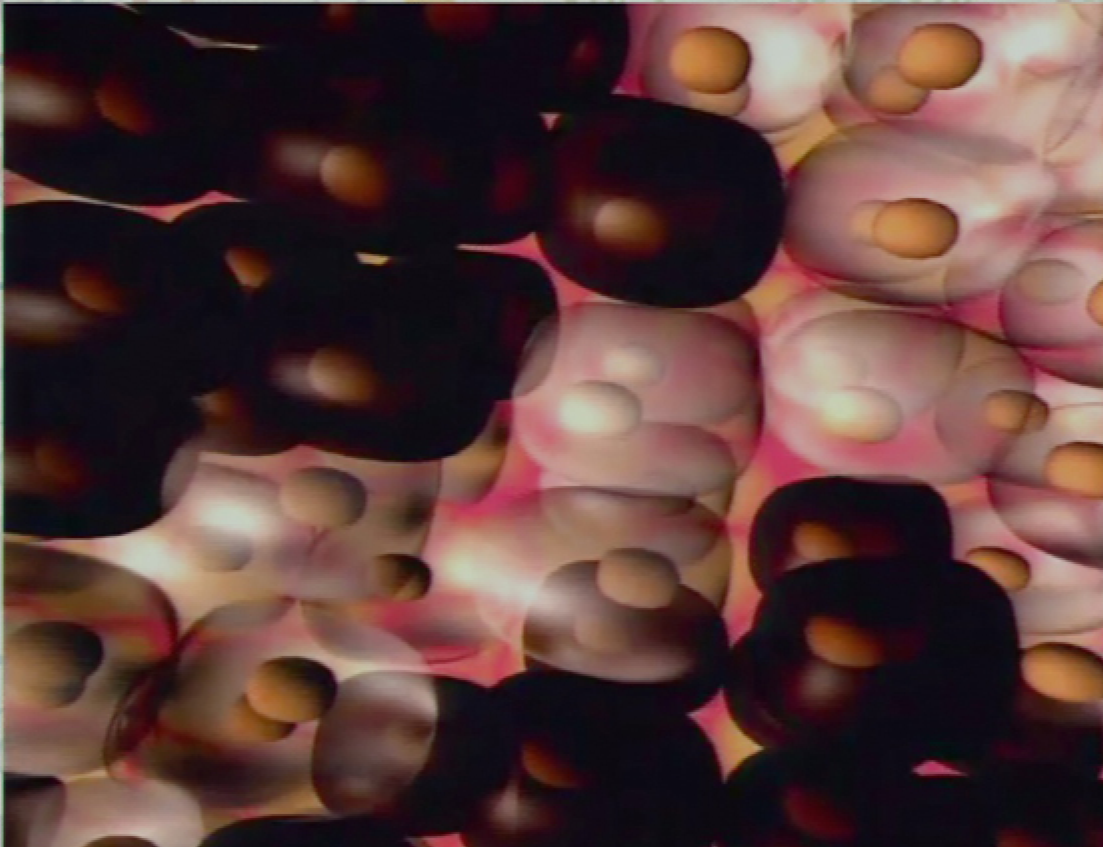
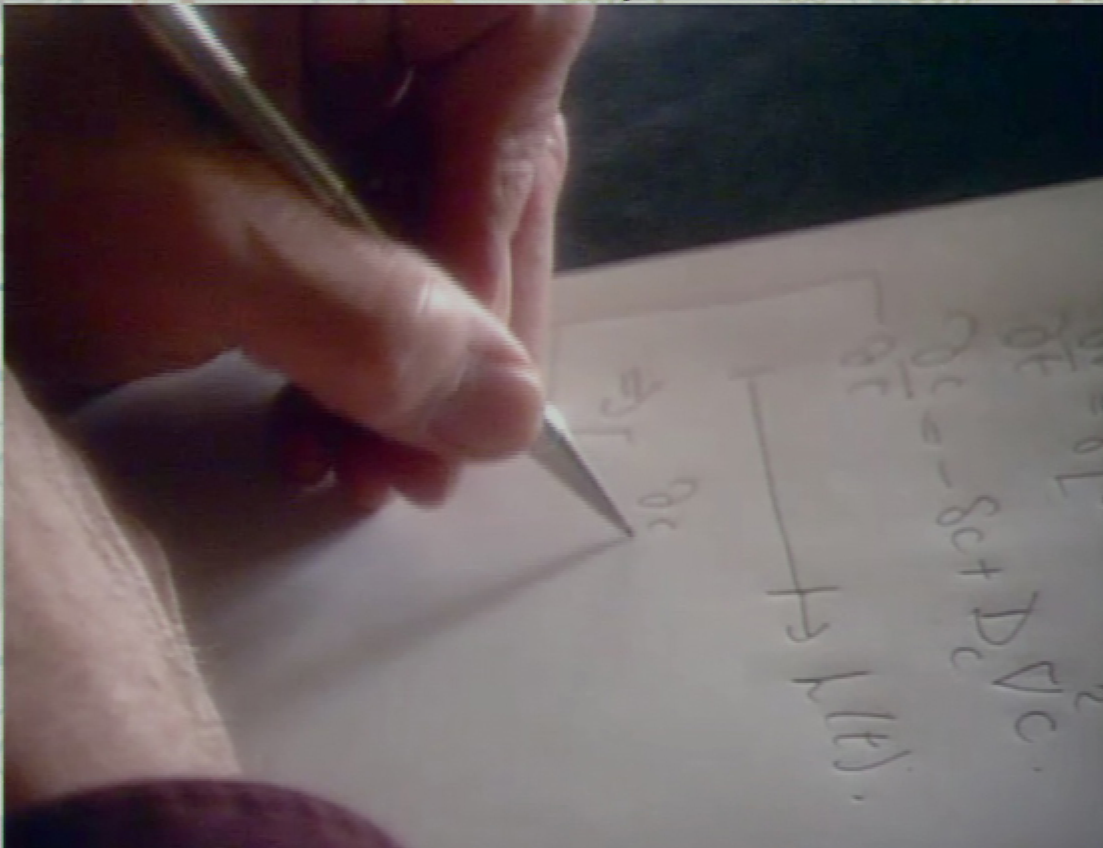


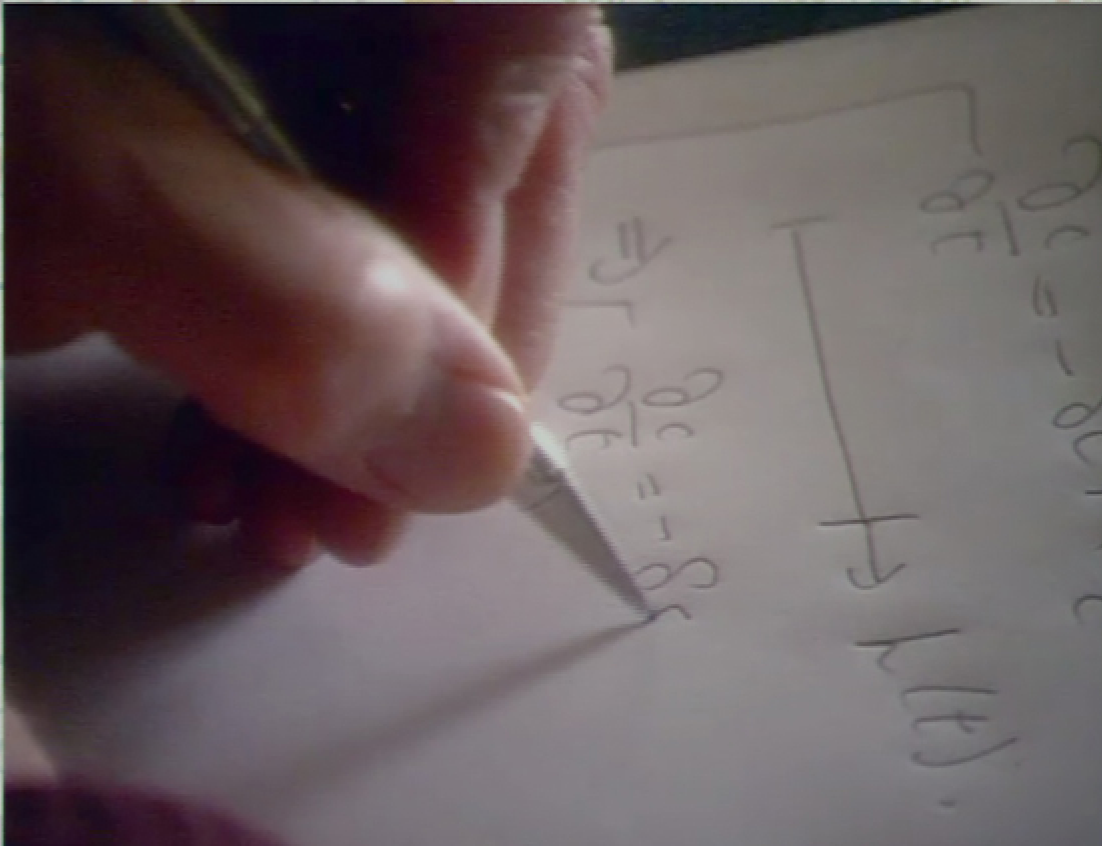
Animal coat patterns

Animal coat patterns

Animal coat patterns



Animal coat patterns



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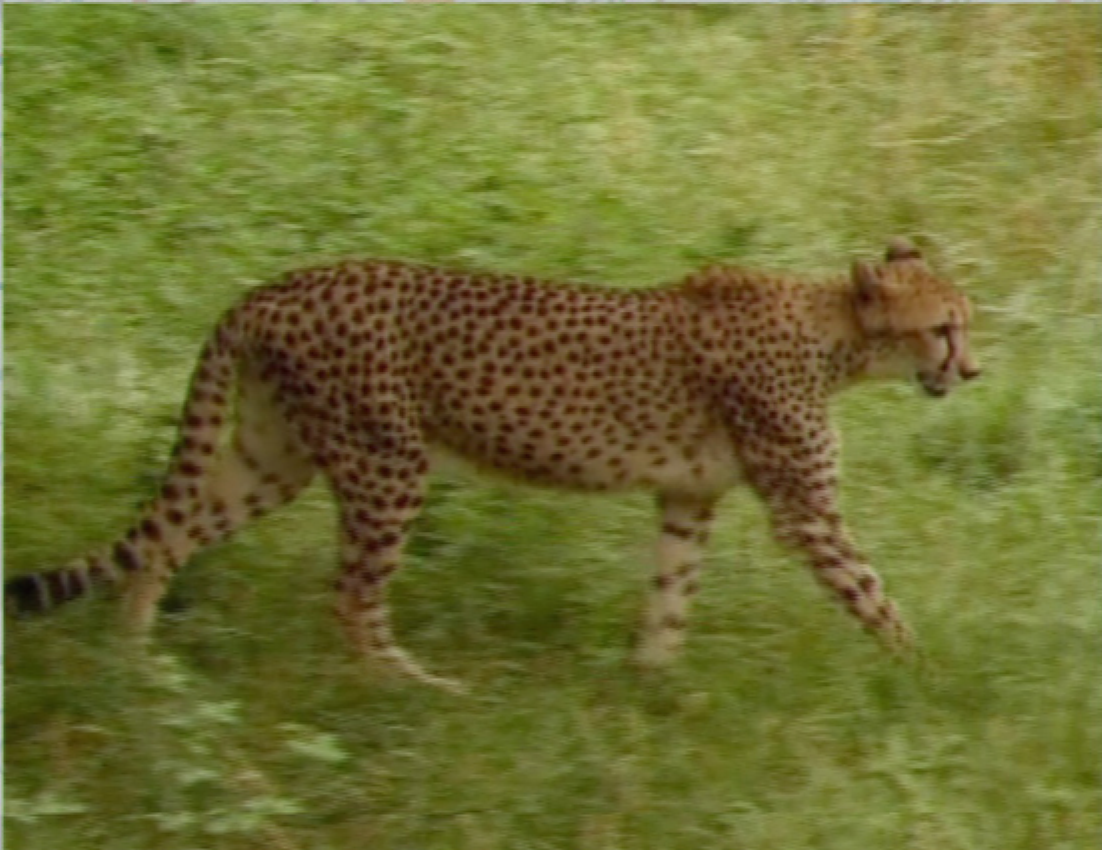
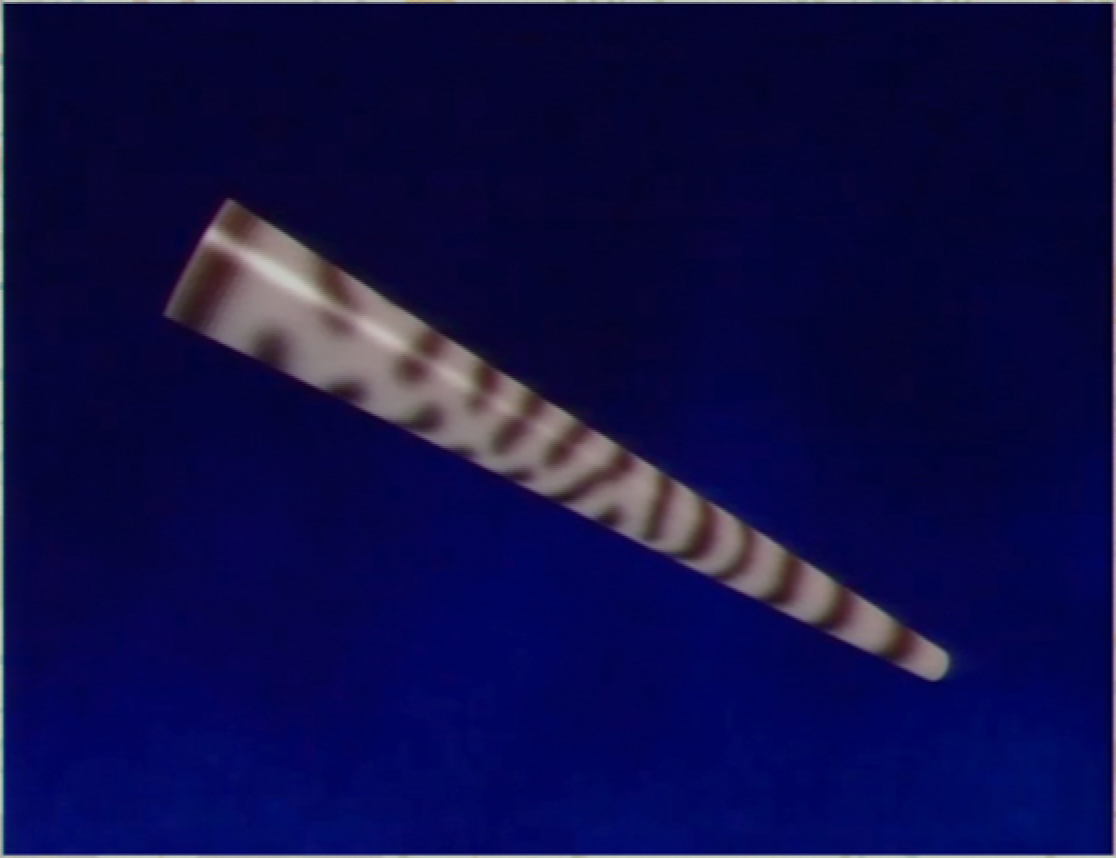
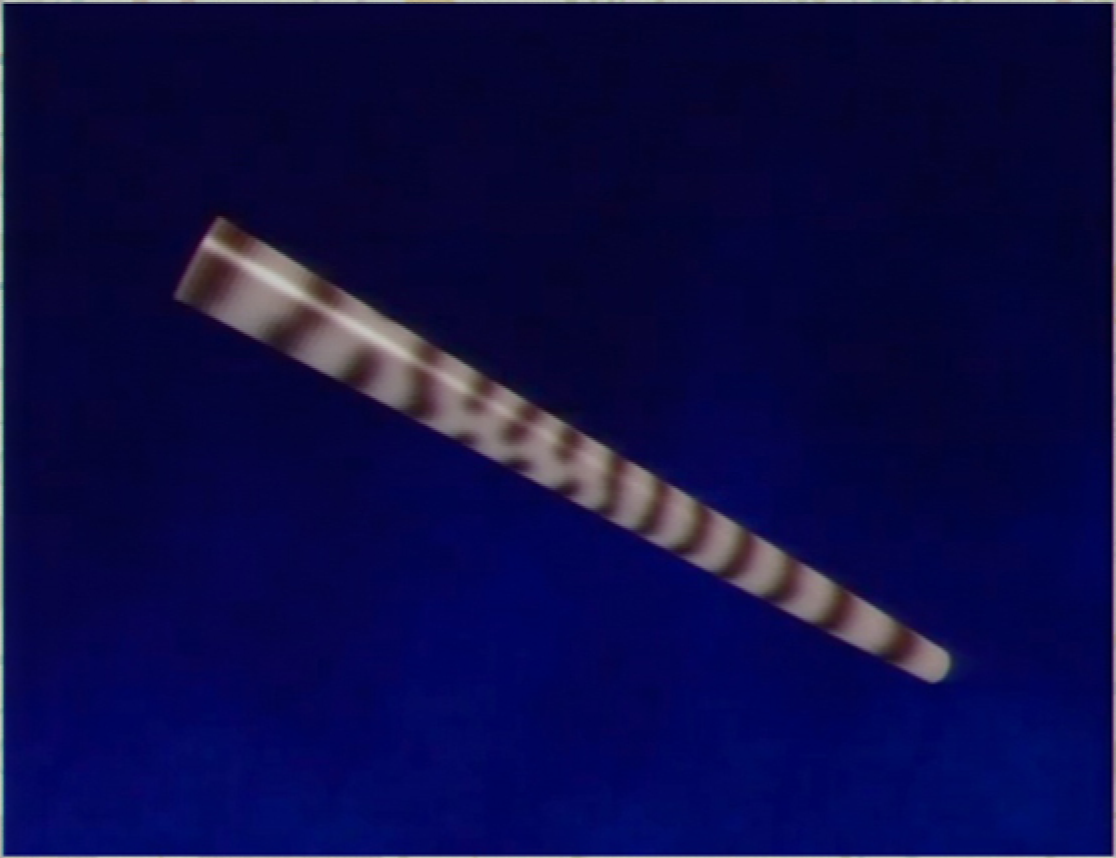


Animal coat patterns



Animal coat patterns



Animal coat patterns

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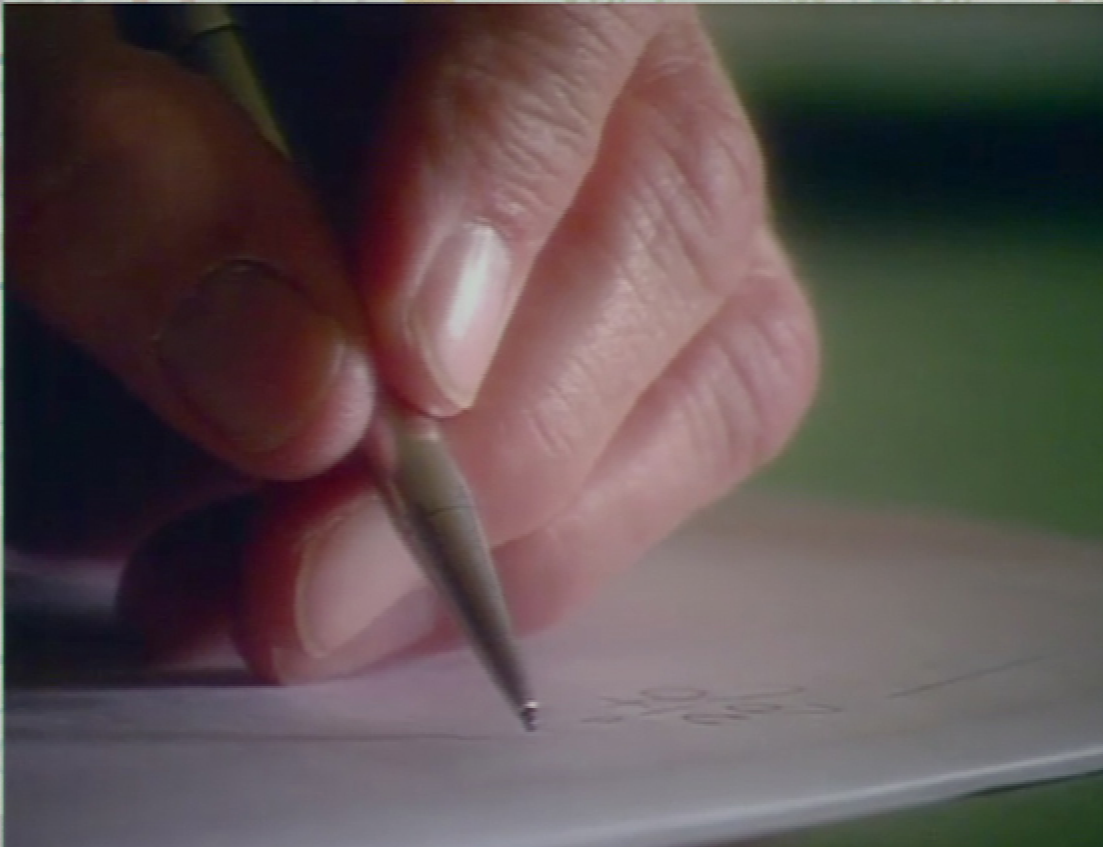

Animal coat patterns

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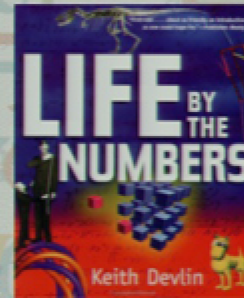
Bodies and tails

Bodies and tails

Murray's computer simulations

MORE BY ME ABOUT THESE IDEAS

Life by the Numbers



Six part television series

PBS: WQED-tv 1998

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http://www.montereymedia.com/science

Companion book published by John Wiley.